

Math 2FM3, Tutorial 11

Dec 2nd, 2015

Term Structure of Interest Rates

- Zero Coupon Bond:
- A zero coupon bond is a bond that has no coupons and has a single payment made at the time of maturity.
- Term Structure:
- The yield to t-year maturity on a 'zero coupon bond' is called the spot rate of interest $s_0(t)$. The set $\{s_0(t)\}_{t>0}$ is the term structure of interest rate.

- Relations between spot rates and yield to maturity.
- If the face amount of the bond is F , coupon rate is r , yield to maturity y_r has
- $$P = Fr[(1+y_r)^{-1} + \dots + (1+y_r)^{-k}] + F(1+r)(1+y_r)^{-(k+1)}$$

$$= Fr[(1+s_0(1))^{-1} + (1+s_0(2))^{-2} + \dots + (1+s_0(k))^{-k}] + F(1+r)(1+s_0(k+1))^{-(k+1)}$$

Forward Rate

- Given the term structure at time, then $n-1$ years forward one-year rate from $n-1$ to n is
- $i_0(n-1,n) = (1+s_0(n))^n / (1+s_0(n-1))^{n-1} - 1$

Ex 6.1.4

- (a) You are given the following information about two 10-year bonds. Both bonds have face amount 100 and coupons payable semi-annually, with next coupon due in $\frac{1}{2}$ -year.
Bond 1: Coupon rate 4% per year, price 85.12.
Bond 2: Coupon rate 10% per year, price 133.34.
Find the yield rate for a 10-year zero coupon bond.
- (b) You are given the following term structure (effective annual interest rates) for zero coupon bond maturities up to n years; $s_0(1) = s_0(2) = \dots = s_0(n-1) < s_0(n)$ (flat term structure except for n -year maturity). An n -year bond has annual coupon rate $r > 0$ and annual coupons. Show that the yield to maturity for the bond j must satisfy $s_0(n-1) < j < s_0(n)$.

Ex 6.1.5

- You are given the following information for 4 bonds. All coupon and yield-to-maturity rates are nominal annual convertible twice per year.

Bond	Time to Maturity	Coupon Rate	YTM
1	½-year	4%	0.05
2	1-year	6%	0.10
3	1.5-year	4%	0.15
4	2-year	8%	0.15

Find the associated term structure for zero coupon bonds with maturities of ½-year, 1-year, 1.5-year and 2-year (quotations should be nominal annual rates convertible twice per year).